Unit-V

Test of significance

- T- test

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$ where n = sample size

$$\bar{x} = \frac{\Sigma x}{n}$$

μ = Population mean from which the sample is taken

S.D ⇒ $s = \sqrt{\frac{\Sigma(x-\bar{x})}{n-1}}$

n-1 = Degree of freedom.

Note:

→ If calculated value of $|t| \geq$ table value for n-1 degree of freedom then reject the hypothesis.

→ If the calculated value of $|t| <$ table value for n-1 degree of freedom then accept the hypothesis.

Q. 9 patients ~~of~~ to whom a certain drink was administrated registered the following increase in their B.P 7, 3, −1, 4, −3, 5, 6, −4, 1. The data indicates that the drink was responsible for this increments (Given that the value for 8 degree of freedom 5% level = 2.71)

Sol:- Given that the value for 8 degree of freedom at 5% level = 2.71

Hypothesis:-

Assume that the drink is responsible for increasing the blood pressure.

The mean of the population is $\mu = 0$ & $n = 9$

x	$x-\bar{x}$	$(x-\bar{x})^2$
7	5	25
3	1	1
-1	-3	9
4	2	4
-3	-5	25
5	3	9
6	4	16
-4	-6	36
1	-1	1

$\Sigma x = 18$

$\Sigma (x-\bar{x})^2 = 126$

$\bar{x} = \frac{\Sigma x}{n} = \frac{18}{9} = 2$

$S.D = s = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n-1}}$

$\Rightarrow \sqrt{\frac{126}{9-1}}$

$s \Rightarrow \sqrt{\frac{126}{8}}$

$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} \Rightarrow \frac{2-0}{\frac{3.9686}{\sqrt{9}}} \Rightarrow \frac{2\times\sqrt{9}}{3.9686} = \frac{6}{3.9686}$

$\therefore t_{cal} = 1.5120 < 2.71$ (Table value)

Hence, we accept the hypothesis.

Q. Find the students t for the following variable values in the sample of ~~10~~ $-6, -4, -1, -1, 0, 1, 1, 3, 4, 5$ taking the mean of the universe to be zero.

Sol:- The mean of the population $\mu = 0$, $n = 10$.

x	$x-\bar{x}$	$(x-\bar{x})^2$
-6	-6.2	38.44
-4	-4.2	17.64
-1	-1.2	1.44
-1	-1.2	1.44
0	-0.2	0.04
1	0.8	0.64
1	0.8	0.64
3	2.8	7.84
4	3.8	14.44
5	4.8	23.04

$\Sigma x = 2$ $\qquad \Sigma (x-\bar{x})^2 \Rightarrow 105.6$

$$\bar{x} = \frac{\Sigma x}{n} \Rightarrow \frac{2}{10}$$

$$x = 0.2$$

$$S.D \Rightarrow s = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n-1}} \Rightarrow \sqrt{\frac{105.6}{10-1}} = 3.425$$

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} \Rightarrow \frac{0.2-0}{\frac{3.425}{\sqrt{10}}} \Rightarrow \frac{0.2 \times \sqrt{10}}{3.425}$$

$$t = 0.1846.$$

Q. The following 10 members are drawn from a population 4, -2, 8, 6, -1, 0, 14, 7, 10, 5. Test whether the population can be 5 (given value for 90 degree of freedom at 5% level = 2.262)

Sol:- The mean of the population is 5

$\mu = 5, \quad n = 10$

x	$x - \bar{x}$	$(x-\bar{x})^2$
4	-1.1	1.21
-2	-7.1	50.41
8	2.9	8.41
6	0.9	0.81
-1	-6.1	37.21
0	-0.5	0.25
14	8.9	79.21
7	1.9	3.61
10	4.9	24.01
5	-0.1	0.01

$\Sigma x = 51$

$\bar{x} = \frac{51}{10}$

$\bar{x} = 5.1$

$\Sigma(x-\bar{x})^2 = 230.9$

S.D $\Rightarrow$ $s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} \Rightarrow \sqrt{\frac{230.9}{10-1}} \Rightarrow 5.0651$

$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} \Rightarrow \frac{5.1-5}{\frac{5.0651}{\sqrt{10}}} \Rightarrow \frac{0.1\times\sqrt{10}}{5.0651} \Rightarrow 0.06243$

< 2.262

Therefore, we accept the hypothesis.

Chi-Square Test of Significance

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

where

O_i = observed frequencies

E_i = Expected frequencies

χ^2 = $(n-1)$ degree of freedom at 5% level.

Q. Find the value of χ^2 of the following data Test the goodness of fit.

observed frequencies:-	14	15	18	20	15	10
Expected frequencies:-	17	10	15	25	10	15

degree of freedom at 5% level is 11.07

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
14	17	-3	9	0.529
15	10	5	25	2.5
18	15	3	9	0.6
20	25	-5	25	1
15	10	5	25	2.5
10	15	-5	25	1.666

$$\frac{\Sigma (O_i - E_i)^2}{E_i} = 8.795$$

$\chi^2_{cal} = 8.795 < 11.07$ (table value)

Therefore, we accept the hypothesis.

Q. The following table gives the no. of air crafts accidents that occured during various days of the week. find whether the accidents are uniformly distributed over the week.

(The value of χ^2 0.05 for degree of freedom is 12.59)

Days:	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents:	14	16	8	12	11	9	14

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2/E_i$
14	12	2	4	0.333
16	12	4	16	1.333
8	12	-4	16	1.333
12	12	0	0	0
11	12	-1	1	0.0833
9	12	3	9	0.75
14	12	2	4	0.333

$$\frac{\sum (O_i - E_i)^2}{E_i} = 4.1653$$

$$\chi^2_{cal} = 4.1653 < 12.59 \text{ (Table value)}$$

Therefore, we accept the Hypothesis.

F-Test (for equality of population variances)

$$f = \frac{s_1^2}{s_2^2} \Rightarrow \text{if} (s_1^2 > s_2^2)$$

or

$$f = \frac{s_2^2}{s_1^2} \Rightarrow \text{if} (s_2^2 > s_1^2)$$

where $s_1^2 = \dfrac{\Sigma(x-\bar{x})^2}{n_1 - 1}$

$$s_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1}$$

- f_{cal} < Table value (Accept the Hypothesis)
- f_{cal} > Table value(Reject the Hypothesis)

Q. Two random samples drawn from two Normal population are

x: sample1:-	20	16	26	27	23	22	18	24	25	19	-	-
y: sample2:-	27	33	42	35	32	34	38	28	41	43	30	37

Obtain the estimate of variance of the 2 populations having the same variance (f for 11 and 9 d.f at 5% level = 3.11)

x	y	$(x-\bar{x})$	$(x-\bar{x})^2$	$(y-\bar{y})$	$(y-\bar{y})^2$
20	27	-2	4	-8	64
16	33	-6	36	-2	4
26	42	4	16	7	49
27	35	5	25	0	0
23	32	1	1	-3	9
22	34	0	0	-1	1
18	38	-4	16	3	9
24	28	2	4	-7	49
25	41	3	9	6	36
19	3·43	-3	9	8	64
—	30	-	-	-5	25
—	37	-	-	2	4

$$s_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1} \Rightarrow \frac{120}{10-1} \rightarrow 13.33$$

$$s_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2-1} = \frac{314}{12-1} \rightarrow 28.54$$

$$s_2^2 > s_1^2$$

$$F_{cal} = \frac{s_2^2}{s_1^2} = \frac{28.54}{13.33} = 2.14$$

$F_{cal} = 2.14 < 3.11$ (Table values)

Therefore, we accept the Hypothesis.

Q: Two samples of size 9 & 8 given the sum of square of deviation from their respective mean = 160 inches sq) & 91 inches sq respectively then they regarded as drawn from the 2 normal population with same variance

(f for 8 & 7 df => 3.73)

Sol:- Given $n_1 = 9$ & $n_2 = 8$

$$\Sigma(x-\bar{x})^2 = 160$$

$$\Sigma(y-\bar{y})^2 = 91$$

$$s_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1} = \frac{160}{9-1} = 20$$

$$s_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1} = \frac{91}{8-1} \Rightarrow 13$$

$$s_1^2 > s_2^2$$

$$F_{cal} = \frac{s_1^2}{s_2^2} \Rightarrow \frac{20}{13} = 1.538$$

$$F_{cal} = 1.538 < 3.73$$

Therefore, we accept the Hypothesis.