

Unit-4 Statistics

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9:38 AM

Curve fitting method by Least squares

- 1) Straight Line
- 2) Parabola [2^{nd} degree]
- 3) Exponential

Straight Line

Working Rule

- Fitting of a straight line be $y = a + bx$ — (1)
- Normal equations are
$$\sum y = na + b \sum x \text{ — (2)}$$
$$\sum xy = a \sum x + b \sum x^2 \text{ — (3)}$$
- Tabular Column

x	y	xy	x^2
$\sum x$	$\sum y$	$\sum xy$	$\sum x^2$

- Substitute the table values in eq (2) & eq (3)
- Solve eq (2) & (3) to get a and b
- Substitute values of a and b in eq (1) to get required straight line

Q. fit a straight line of best fit to the following data by the method of least square

x:	6	7	7	8	8	8	9	9	10
y:	5	5	4	5	4	3	4	3	3

Soln-

x	y	xy	x ²
6	5	30	36
7	5	35	49
7	4	28	49
8	5	40	64
8	4	32	64
8	3	24	64
9	4	36	81
9	3	27	81
10	3	30	100

$$y = a + bx \quad \text{--- (1)}$$

$$\sum y = na + b \sum x \quad \text{--- (2)}$$

$$\sum xy = a \sum x + b \sum x^2 \quad \text{--- (3)}$$

$$\sum x = 72 \quad \sum y = 36 \quad \sum xy = 282 \quad \sum x^2 = 588$$

Substitute the table values in eq (2) & eq (3)

$$36 = 9a + 72b$$

$$282 = 72a + 588b$$

} solve & get a, b

a = 8, b = -0.5 substitute a, b in eq (1)

The equation of straight line of best fit

$$y = 8 - 0.5x$$

Q. Fit a straight line of a best fit to the following data by the method of least square.

$x:$	1	2	3	4	6	8
$y:$	2.4	3	3.6	4	5	6

soln

x	y	xy	x^2
1	2.4	2.4	1
2	3	6	4
3	3.6	10.8	9
4	4	16	16
6	5	30	36
8	6	48	64

$$\Sigma x = 24$$

$$\Sigma y = 24$$

$$\Sigma xy = 113.2$$

$$\Sigma x^2 = 130$$

Substitute the table values in eq ② & eq ③

$$\left. \begin{aligned} 24 &= 6a + 24b \\ 113.2 &= 24a + 130b \end{aligned} \right\} \text{solve \& get } a, b$$

$$a = 1.976 \quad b = 0.505$$

Substitute a, b in eq ①

The equation of straight line of best fit

$$\boxed{y = 1.976 + 0.505x}$$

Parabola { 2nd degree }

Working Rule

- Fitting of a parabola [2nd degree] curve be

$$y = a + bx + cx^2 \text{ --- (1)}$$

- Normal equations are

$$\sum y = na + b\sum x + c\sum x^2 \text{ --- (2)}$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3 \text{ --- (3)}$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4 \text{ --- (4)}$$

- Tabular column

x	y	x ²	x ³	x ⁴	xy	x ² y
$\sum x =$	$\sum y =$	$\sum x^2 =$	$\sum x^3 =$	$\sum x^4 =$	$\sum xy =$	$\sum x^2y =$

- Substitute the table values in eq (2), (3) & (4)
- solve (2), (3), (4) to get the values of a, b, c
- substitute a, b, c in eq (1) to get the required parabola curve

Q. fit a 2nd degree parabola by using least square method approximate for the following data

x:	1	2	3	4
y:	1.7	1.8	2.3	3.2

x	y	x ²	x ³	x ⁴	xy	x ² y
1	1.7	1	1	1	1.7	1.7
2	1.8	4	8	16	3.6	7.2
3	2.3	9	27	81	6.9	20.7
4	3.2	16	64	256	12.8	51.2

$$\Sigma x = 10 \quad \Sigma y = 9 \quad \Sigma x^2 = 30 \quad \Sigma x^3 = 100 \quad \Sigma x^4 = 354 \quad \Sigma xy = 25 \quad \Sigma x^2y = 80.8$$

Substitute the table values in (2), (3), (4)

$$\left. \begin{aligned} 9 &= 4a + 10b + 30c \\ 25 &= 10a + 30b + 100c \\ 80.8 &= 30a + 100b + 354c \end{aligned} \right\} \text{solve to get } a, b, c$$

$$a = 2, \quad b = -0.5, \quad c = 0.2 \quad \text{substitute in eq (1)}$$

The curve of best fit is $y = 2 - 0.5x + 0.2x^2$

Q. fit a parabola by using least square approximate for the following data

x:	1	2	3	4	5
y:	1.2	1.6	3.2	3.6	5.2

x	y	x^2	x^3	x^4	xy	x^2y
1	1.2	1	1	1	1.2	1.2
2	1.6	4	8	16	3.2	6.4
3	3.2	9	27	81	9.6	28.8
4	3.6	16	64	256	14.4	57.6
5	5.2	25	125	625	26	130

$$\sum x = 15, \quad \sum y = 14.8, \quad \sum x^2 = 55, \quad \sum x^3 = 225, \quad \sum x^4 = 979, \\ \sum xy = 54.4, \quad \sum x^2y = 224$$

Substitute table values in eq ②, ③ & ④

$$\begin{aligned} 14.8 &= 5a + 15b + 55c \\ 54.4 &= 15a + 55b + 225c \\ 224 &= 55a + 225b + 979c \end{aligned} \quad \left. \vphantom{\begin{aligned} 14.8 &= 5a + 15b + 55c \\ 54.4 &= 15a + 55b + 225c \\ 224 &= 55a + 225b + 979c \end{aligned}} \right\} \text{solve to get } a, b, c$$

$$a = 0.56, \quad b = 0.485, \quad c = 0.085 \quad \text{Substitute in eq ①}$$

The curve of best fit is

$$y = 0.56 + 0.485x + 0.085x^2.$$

Exponential curve

Working Rule

• Fitting of a curve of best fit $y = ae^{bx}$ — ①

Take log on both sides

$$\log y = \log ae^{bx}$$

$$\log y = \log a + \log e^{bx}$$

$$\log y = \log a + bx \log e$$

$$Y = A + Bx$$

where $\log y = Y$, $\log a = A$, $b \log e = B$

- Normal equations are

$$\sum Y = nA + B \sum x \quad \text{--- (2)}$$

$$\sum xY = A \sum x + B \sum x^2 \quad \text{--- (3)}$$

- Tabular column

x	y	$Y = \log y$	x^2	xY

- Substitute table values in eq (2) & (3) to get A & B .
- Then find a & b
- Substitute a, b in equation (1) to the best fit curve.

Q1] fit a curve of best fit of the type $y = ae^{bx}$ to the following data

$x:$	1	5	7	9	12
$y:$	10	15	12	15	21

x	y	$Y = \log y$	x^2	xY
1	10	1.0000	1	1.000
5	15	1.1760	25	5.88
7	12	1.0791	49	7.5537
9	15	1.1760	81	10.584
12	21	1.32222	144	15.8664

$$\sum x = 34$$

$$\sum y = 73$$

$$\sum xy = 5.7533$$

$$\sum x^2 = 300$$

$$\sum xy = 40.8841$$

Substitute table values in eq (2) & (3)

$$5.7533 = 5A + 34B$$

$$40.8841 = 34A + 300B$$

$$A = 0.9765, B = 0.0256$$

$$A = \log a$$

$$\text{Anti} \cdot \log A = a$$

$$\text{Anti } 0.9765 = a$$

$$a = 9.4754$$

$$b \log e = B$$

$$b = \frac{B}{\log e} = \frac{0.0256}{\log(2.718)} = b = 0.0584$$

Substitute a, b in eq (1)

The curve of best fit is $y = 9.4754 e^{0.0584x}$ //

Q1) fit a curve of best fit of the type $y = ae^{bx}$ to the following data

$x:$	1	2	3	4	5
$y:$	1.2	1.6	1.8	2	2.2

x	y	$Y = \log y$	x^2	xy
1	1.2	0.0791	1	0.0791
2	1.6	0.2041	4	0.4082
3	1.8	0.2552	9	0.7656
4	2	0.3010	16	1.204
5	2.2	0.3424	25	1.712

$$\sum x = 15$$

$$\sum y = 8.8$$

$$\sum Y = 1.1818$$

$$\sum x^2 = 55$$

$$\sum xy = 4.1689$$

Substitute table values in
eq (2) & eq (3)

$$1.1818 = 5A + 15B$$

$$4.1689 = 15A + 55B$$

$$A = 0.04931 \quad B = 0.06235$$

$$A = \log a$$

$$\text{Anti } \log A = a$$

$$\text{Anti} \cdot 0.04931 = a$$

$$\boxed{a = 1.1202}$$

$$\log b = B$$

$$b = \frac{B}{\log e} = \frac{0.06235}{\log(2.718)} \Rightarrow \frac{0.06235}{0.4377}$$

$$\boxed{b \Rightarrow 0.1424}$$

Substitute a, b in eq (1)

The curve of best fit is $y = 1.1202e^{0.1424x}$

Karl Pearson co-relation coefficient
(Imp)

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$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

$$\boxed{-1 \leq r \leq 1}^{**}$$

Tabular column:-

x	y	x^2	y^2	xy
$\sum x$	$\sum y$	$\sum x^2$	$\sum y^2$	$\sum xy$

Q) Find the correlation coefficient of for the following data

x:	1	2	3	4	5	6
y:	6	5	4	3	2	1

Soln

x	y	x^2	y^2	xy
1	6	1	36	6
2	5	4	25	10
3	4	9	16	12
4	3	16	9	12
5	2	25	4	10
6	1	36	1	6

$$\begin{array}{l|l|l} \Sigma x = 21 & \Sigma x^2 = 91 & \Sigma xy = 56 \\ \Sigma y = 21 & \Sigma y^2 = 91 & \end{array}$$

$$r = \frac{6 \times 56 - 21^2}{\sqrt{6 \times 91 - (21)^2} \cdot \sqrt{6 \times 91 - (21)^2}}$$

$$r = \frac{-105}{\sqrt{105} \sqrt{105}} \Rightarrow \frac{-105}{105}$$

$$\boxed{r = -1} //$$

Artwork by Akram
 "I'd love to hear what inspired your work."

Q) find the correlation coefficient of for the following data.

x:	16	18	20	22	24	26	27	30
y:	25	30	44	51	45	61	45	70

solⁿ

x	y	x^2	y^2	xy
16	25	256	625	400
18	30	324	900	540
20	44	400	1936	880
22	51	484	2601	1122
24	45	576	2025	1080
26	61	676	3721	1586
27	45	729	2025	1215
30	70	900	4900	2100

$$\begin{array}{l|l|l} \sum x = 183 & \sum x^2 = 4345 & \sum xy = 8923 \\ \sum y = 371 & \sum y^2 = 18733 & \end{array}$$

$$r = \frac{8 \times 9067 - 183 \times 371}{\sqrt{8 \times 4345 - (183)^2} \sqrt{8 \times 18733 - (371)^2}}$$

$$r = \frac{3491}{\sqrt{1271} \sqrt{12223}}$$

$$r = \frac{3491}{3941.501} \Rightarrow \boxed{0.8858}$$

Q) Find the correlation coefficient of for the following data

x:	68	64	75	50	64	80	75	40	55	64
y:	62	58	68	45	81	60	68	48	50	70

Solⁿ

x	y	x^2	y^2	xy
68	62	4624	3844	4216
64	58	4096	3364	3712
75	68	5625	4624	5100
50	45	2500	2025	2250
64	81	4096	6561	5184
80	60	6400	3600	4800
75	68	5625	4624	5100
40	48	1600	2304	1920
55	50	3025	2500	2750
64	70	4096	4900	4480

$$\begin{array}{l|l|l} \Sigma x = 635 & \Sigma x^2 = 41687 & \\ \Sigma y = 610 & \Sigma y^2 = 38346 & \Sigma xy = 39515 \end{array}$$

$$r = \frac{10 \times 39515 - 635 \times 610}{\sqrt{10 \times 41687 - (635)^2} \sqrt{10 \times 38346 - (610)^2}}$$

$$\Rightarrow \frac{7800}{\sqrt{13645} \sqrt{11360}}$$

$$\Rightarrow \frac{7800}{12450.18875} \Rightarrow 0.6264$$

$$\Rightarrow 0.6264$$

Lines of Regression (Imp):-

- Regression equation of x on y

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$\Rightarrow x - \bar{x} = b_{xy} (y - \bar{y})$$

where $b_{xy} = r \frac{\sigma_x}{\sigma_y} \Rightarrow \frac{\sum xy}{\sum y^2}$; $\bar{x} = \frac{\sum x}{n}$, $\bar{y} = \frac{\sum y}{n}$

b_{xy} is the regression coefficient of x on y

- Regression equation of y on x

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$\Rightarrow y - \bar{y} = b_{yx} (x - \bar{x})$$

where $b_{yx} = r \frac{\sigma_y}{\sigma_x} = \frac{\sum xy}{\sum x^2}$; $\bar{x} = \frac{\sum x}{n}$; $\bar{y} = \frac{\sum y}{n}$

$$r = \sqrt{(b_{xy})(b_{yx})}$$

a) Calculate the coefficient of correlation & obtain the least square regression lines for the following data

x:	1	2	3	4	5	6	7	8	9
y:	9	8	10	12	11	13	14	16	15

Soln-

x	y	x^2	y^2	xy
1	9	1	81	9
2	8	4	64	16
3	10	9	100	30
4	12	16	144	48
5	11	25	121	55
6	13	36	169	78
7	14	49	196	98
8	16	64	256	128
9	15	81	225	135

$$\begin{array}{l|l} \Sigma x = 45 & \Sigma x^2 = 285 \\ \Sigma y = 108 & \Sigma y^2 = 1356 \end{array} \quad \Sigma xy = 597$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{45}{9} = \boxed{5} \quad b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{597}{1356} = \boxed{0.440}$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{108}{9} = \boxed{12} \quad b_{yx} = \frac{\Sigma xy}{\Sigma x^2} = \frac{597}{285} = \boxed{2.0947}$$

$$r = \sqrt{(b_{xy})(b_{yx})} = \sqrt{(0.440)(2.0947)}$$

$$r \Rightarrow \sqrt{0.921668}$$

$$\boxed{r = 0.9600}$$

Regression lines of x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 5 = (0.440)(y - 12)$$

$$x - 5 = 0.44y - 5.28$$

$$x = 0.44y - 5.28 + 5$$

$$\boxed{x = 0.44y - 0.28}$$

Regression line of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 12 = 2.094(x - 5)$$

$$y - 12 = 2.094x - 10.4735$$

$$y = 2.094x - 10.4735 + 12$$

$$\boxed{y = 2.094x + 1.52}$$

Q) Calculate the coefficient of correlation & obtain the least square regression lines for the following data.

x:	1	2	3	4	5	6	7	8	9	10
y:	10	12	16	28	25	36	41	49	40	50

Solⁿ →

x	y	x^2	y^2	xy
1	10	1	100	10
2	12	4	144	24
3	16	9	256	48
4	28	16	784	112
5	25	25	625	125
6	36	36	1296	216
7	41	49	1681	287
8	49	64	2401	392
9	40	81	1600	360
10	50	100	2500	500

$$\begin{array}{l|l|l} \Sigma x = 55 & \Sigma x^2 = 385 & \Sigma xy = 2074 \\ \Sigma y = 307 & \Sigma y^2 = 11387 & \end{array}$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{55}{10} = 5.5 \quad \left| \quad b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{2074}{11387} = \boxed{0.1821} \right.$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{307}{10} = 30.7 \quad \left| \quad b_{yx} = \frac{\Sigma xy}{\Sigma x^2} = \frac{2074}{385} = \boxed{5.3870} \right.$$

$$r = \sqrt{(b_{xy})(b_{yx})} = \sqrt{(0.1821)(5.3870)}$$

$$r \Rightarrow \sqrt{0.9809}$$

$$r = 0.9904$$

Regression line of x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 5.5 = 0.1821(y - 30.7)$$

$$x = 0.1821y - 5.5904 + 5.5$$

$$x = 0.1821y - 0.0904$$

Regression line of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 30.7 = 5.3870(x - 5.5)$$

$$y = 5.3870x - 29.6285 + 30.7$$

$$y = 5.3870x + 1.0715$$

Q. Obtain the lines of regression y on x & x on y of the following data.

$x:$	1	2	3	4	5
$y:$	5	7	9	10	11

Solⁿ:-

x	y	x^2	y^2	xy
1	5	1	25	5
2	7	4	49	14
3	9	9	81	27
4	10	16	100	40
5	11	25	121	55

$$\sum x = 15$$

$$\sum x^2 = 55$$

$$\sum xy = 141$$

$$\sum y = 42$$

$$\sum y^2 = 376$$

$$\bar{x} = \frac{\sum x}{n} = \frac{15}{5} = \boxed{3}$$

$$\bar{y} = \frac{\sum y}{n} = \frac{42}{5} = \boxed{8.4}$$

$$b_{xy} = \frac{\sum xy}{\sum y^2} = \frac{141}{376} = \boxed{0.375}$$

$$b_{yx} = \frac{\sum xy}{\sum x^2} = \frac{141}{55} = \boxed{2.563}$$

$$r = \sqrt{(b_{xy})(b_{yx})}$$

$$\Rightarrow \sqrt{(0.375)(2.563)} \Rightarrow \sqrt{0.961125}$$

$$\boxed{r = 0.9803}$$

Regression line of x on y :-

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$x - 3 = 0.375(y - 8.4)$$

$$x-3 = 0.375y - 3.15$$

$$x = 0.35$$

$$x = 0.375y - 3.15 + 3$$

$$x = 0.375y - 0.15$$

Regression Line of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 8.4 = 2.563(x - 3)$$

$$y - 8.4 = 2.563x - 7.689$$

$$y = 2.563x - 7.689 + 8.4$$

$$y = 2.563x + 0.711$$

Imp
Q. In a partially destroyed laboratory a record of Analysis of Correlation data the following results are only visible
variance is 9, the regression $8x - 10y + 66 = 0$ & $40x - 18y + 214 = 0$. Find the means of x, y coefficient of correlation b/w x & y and also the standard deviation of y.

Sol:- Given:-

$$8x - 10y + 66 = 0 \quad \text{--- (1)}$$

$$40x - 18y + 214 = 0 \quad \text{--- (2)}$$

eq (1) $\times 5$

$$40x - 50y + 330 = 0$$

$$40x - 18y - 214 = 0$$

$$\hline -32y + 544 = 0$$

$$-32y = -544$$

$$y = \frac{-544}{-32}$$

$$\boxed{y = 17}$$

Substitute in eq^①

$$8x - 10y + 66 = 0$$

$$8x - 10(17) + 66 = 0$$

$$8x = 104$$

$$x = \frac{104}{8}$$

$$\boxed{x = 13}$$

Let us suppose the line y on x

$$8x - 10y + 66 = 0$$

$$10y = 8x + 66$$

$$\Rightarrow y = \frac{8}{10}x + \frac{66}{10}$$

$$\boxed{y = 0.8x + 6.6}$$

$\Rightarrow$

$$\boxed{b_{yx} = 0.8}$$

Let us suppose the line x on y

$$40x - 18y - 214 = 0$$

$$40x = 18y + 214$$

$$x = \frac{18}{40}y + \frac{214}{40}$$

$$\boxed{x = 0.45y + 5.35}$$

$\Rightarrow$

$$\boxed{b_{xy} = 0.45}$$

Coefficient of co-relation

$$r = \sqrt{b_{xy} \times b_{yx}}$$

$$\boxed{r = 0.6}$$

$$\text{Variance} = 9$$

$$\sigma_x = \sqrt{9} = 3 \Rightarrow \sqrt{\text{Variance}}$$

$$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}$$

$$\sigma_y = \frac{r \cdot \sigma_x}{b_{xy}} \Rightarrow \frac{0.6 \times 3}{0.45}$$

$$\sigma_y = 4$$

Standard deviation of $y = 4$.

Spearman's Rank of Co-relation

Note:-

$$\rightarrow r = 1 - \frac{6 \sum d^2}{n(n^2-1)}$$

$$\rightarrow r = 1 - \frac{6 \left[\sum d^2 + \frac{1}{12} m_1(m_1-1) + \frac{1}{12} m_2(m_2-1) + \frac{1}{12} m_3(m_3-1) + \dots \right]}{n(n^2-1)}$$

$$\rightarrow 0 < r < 1$$

1. Calculate the spearman's rank coefficient

$x:$	10	15	12	17	13	16	24	14	22
$y:$	30	42	45	46	33	34	40	35	39

x	y	Rank x_i	Rank y_i	$d = x_i - y_i$	d^2
10	30	9	9	0	0
15	42	5	3	2	4
12	45	8	2	6	36
17	46	3	1	2	4
13	33	7	8	-1	1
16	34	4	7	-3	9
24	40	1	4	-3	9
14	35	6	6	0	0
22	39	2	5	-3	9

$$r = 1 - \frac{6 \sum d^2}{n(n^2-1)} \Rightarrow 1 - \frac{6 \times 72}{9(9^2-1)}$$

$$\Rightarrow 1 - \frac{432}{9(81-1)} \Rightarrow 1 - \frac{432}{720}$$

$$r = 0.4$$

2. Obtain the rank & co-relation b/w the variables X & Y , from the following data.

$X:$	20	25	40	20	25	35	20	40	50	60
$Y:$	200	200	230	250	285	230	270	245	230	295

X	Y	Rank X_i	Rank Y_i	$d = X_i - Y_i$	d^2
20	200	9	9.5	-0.5	0.25
25	200	6.5	9.5	-3	9
40	230	3.5	7	-3.5	12.25
20	250	9	4	5	25
25	285	6.5	2	4.5	20.25
35	230	5	7	-2	4
20	270	9	3	6	36
40	245	3.5	5	-1.5	2.25
50	230	8	7	-5	25
60	295	1	1	0	0

{I} 2 or 3 X, Y are equal, take rank as average

$$r = \frac{1 - 6 \left[\sum d^2 + \frac{1}{12} m_1(m_1-1) + \frac{1}{12} m_2(m_2-1) + \dots \right]}{n(n^2-1)}$$

$$r = \frac{1 - 6 \left[134 + \frac{6}{12} + \frac{2}{12} + \frac{2}{12} + \frac{6}{12} + \frac{2}{12} \right]}{990}$$

$$\Rightarrow 1 - \frac{814}{990} \Rightarrow \boxed{r = 0.1788}$$

Q. obtain the rank co-relation coefficient for the following data

$x:$	68	64	75	50	64	80	75	40	65	64
$y:$	62	58	68	45	81	60	68	48	50	70

x	y	Rank in x_i	Rank in y_i	$d = x_i - y_i$	d^2
68	62	4	5	-1	1
64	58	7	7	0	0
75	68	2.5	3.5	-1	1
50	45	9	10	-1	1
64	81	7	1	6	36
80	60	1	6	-5	25
75	68	2.5	3.5	-1	1
40	48	10	9	1	1
65	50	5	8	-3	9
64	70	7	2	5	25

$$r = 1 - \frac{6 \left[\sum d^2 + \frac{1}{12} m_1(m_1-1) + \frac{1}{12} m_2(m_2-1) + \dots \right]}{n(n^2-1)}$$

$$r = \frac{1 - 6 \left[100 + \frac{1}{12} 3(3-1) + \frac{1}{12} 2(2-1) + \frac{1}{12} 2(2-1) \right]}{5(5^2-1) - 10(10^2-1)}$$