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Unit-I Probability

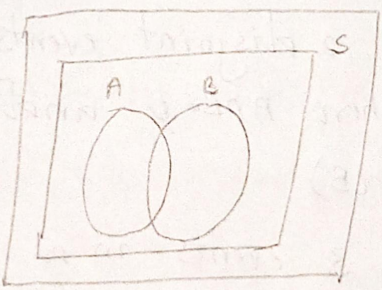
Classical definition of probability:-

There are 'n' exhaustive equally likely elementary events in a try and 'm' of them are favourable to an event A. Then $\frac{m}{n}$ is called Probability of A. It is denoted by P(A)

$$P(A) = \frac{\text{Number of events favourable to A}}{\text{Total number of events}} = \frac{m}{n}$$

Addition Theorem of Probability (or) Theorem of Total probability:-

Statement:- If A and B are two events in a sample space 'S' then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



Note:- Addition Theorem & Total probability Theorem are 2 different Theorems.

Proof:-

* Case 1:- Suppose $A \cap B = \phi$

From definition of probability

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Case 2:- Suppose $A \cap B \neq \phi$

From diagram

$B = (B-A) \cup (A \cap B)$ and $(B-A) \cap (A \cap B) = \phi$

$P(B) = P[(B-A) \cup (A \cap B)]$

$P(B) = P(B-A) + P(A \cap B) - P[(B-A) \cap (A \cap B)]$

$P(B) \Rightarrow P(B-A) + P(A \cap B)$

$P(B-A) = P(B) - P(A \cap B)$

also, $A \cup B = A \cup (B-A)$ and $A \cap (B-A) = \phi$

$P(A \cup B) = P[A \cup (B-A)]$

$P(A \cup B) \Rightarrow P(A) + P(B-A) - P[A \cap (B-A)]$

$P(A \cup B) = P(A) + P(B-A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note:-

→ If A and B are 2 disjoint events in a sample space S then $A \cap B = \phi$ and

$P(A \cup B) = P(A) + P(B)$

→ If A, B, C are 3 events in a sample space S then

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

→ If A & B are 2 events in a sample space S then $P(B-A) = P(B) - P(B \cap A)$

$P(A-B) = P(A) - P(A \cap B)$

Q] When 3 fair coins are tossed simultaneously find the probability to get atleast one tail.

Sol:- Total number of events $= 2 \times 2 \times 2 = 8$

Let $\bar{A}$ be an event to get 3 heads $= 1$

$$P(\bar{A}) = \frac{\text{No. of favourable event}}{\text{Total no. of events}} = \frac{1}{8}$$

Probability of getting at least one tail

$$P(A) = 1 - P(\bar{A}) = 1 - \frac{1}{8} = \frac{7}{8}.$$

Q] Find the probability of getting a number greater than 2 when a dice is rolled.

Sol:- Total number of events when a dice is rolled $= 6 \Rightarrow [1, 2, 3, 4, 5, 6]$

Let 'A' be an event of getting a number greater than $2 = \{3, 4, 5, 6\} - 4$

$\therefore$ favourable events $= 4$

$\therefore$ Required $P(A) = \frac{\text{No. of favourable events to A}}{\text{Total number of events}}$

$$\Rightarrow \frac{4}{6} = \frac{2}{3}$$

Q) A bag contains 4 red, 5 black, 6 blue balls what is the probability that 2 balls drawn simultaneously are one red & one black.

Sol:- Total no. of balls = 4+5+6 = 15 balls

Total number of events = $^{15}C_2 = 105$

Let 'A' be an event of getting one red & one black ball

$\Rightarrow$ No. of favourable event

$\Rightarrow$ $^4C_1 \times {}^5C_1 = 4 \times 5 = 20$

$$P(A) = \frac{\text{No. of favourable event to A}}{\text{Total no. of events}} = \frac{20}{105} = \frac{4}{21}$$

6/4/22 | 10:30AM **Addition Theorem Problems:-**

Q) The probability for a contractor to get a road contract is $\frac{2}{3}$ and building contract is $\frac{5}{9}$. The probability to get atleast one contract is $\frac{4}{5}$. find the probability that he gets both the contracts.

Sol:- Let "A" be an event to get road contract

Let "B" be an event to get building contract

$P(A) = \frac{2}{3}$, $P(B) = \frac{5}{9}$, $P(A \cup B) = \frac{4}{5}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{4}{5} = \frac{2}{3} + \frac{5}{9} - P(A \cap B)$

$P(A \cap B) = \frac{2}{3} + \frac{5}{9} - \frac{4}{5} \Rightarrow \frac{19}{45}$

Q) A, B, C are 3 newspaper published for a city. 20% of the population reads A, 16% of the population reads B, 14% reads C, 8% reads both A and B, 5% reads both B and C, 4% reads A and C, 2% reads all A, B, C. find the percentage of population to read atleast one newspaper.

Sol: Let A, B, C are 3 events of reading news-paper

$P(A) = \frac{20}{100}$, $P(B) = \frac{16}{100}$, $P(C) = \frac{14}{100}$, $P(A \cap B) = \frac{8}{100}$,

$P(B \cap C) = \frac{5}{100}$, $P(A \cap C) = \frac{4}{100}$, $P(A \cap B \cap C) = \frac{2}{100}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{20}{100} + \frac{16}{100} + \frac{14}{100} - \frac{8}{100} - \frac{5}{100} - \frac{4}{100} + \frac{2}{100}$$

$$\Rightarrow \frac{35}{100} \Rightarrow 35\%$$

$\therefore$ Percentage of the population who reads atleast one newspaper is 35

a) If one ticket is randomly selected from a ticket numbers 1 to 30 then find the probability that the number on the ticket is

i) A multiple of 5 or 7

ii) A multiple of 3 or 5

Sol: Total no. of Tickets (Total no. of events) = 30

Let A, B, C are events which are multiple of 5, 7, 3 i.e

$A = \{5, 10, 15, 20, 25, 30\}$

$B = \{7, 14, 21, 28\}$

$C = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$

$P(A) = \dfrac{\text{No. of favourable events to } A}{\text{Total number of events}} = \dfrac{6}{30}$

$P(B) = \dfrac{4}{30}$, $P(C) = \dfrac{10}{30}$

$P(A \cap B) = \dfrac{0}{30} = 0$ $P(A \cap C) = \dfrac{2}{30}$

i) The probability that the number is multiple of 5, 7

$P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow \dfrac{6}{30} + \dfrac{4}{30} - 0 = \dfrac{10}{30} = \dfrac{1}{3}$

ii) The probability that the number is a multiple of 3, 5

$P(A \cup C) = P(A) + P(B) - P(A \cap C)$

$P(A \cup C) = \dfrac{6}{30} + \dfrac{10}{30} - \dfrac{2}{30} \Rightarrow \dfrac{14}{30} = \dfrac{7}{15}$

Q) A bag contains 4 red, 3 blue, balls. A ball is drawn randomly from the bag. find the probability of getting either red or blue ball.

Solⁿ:- Total number of balls = 7

Number of red balls = 4

Number of blue balls = 3

Probability of getting a red ball $P(A) = \frac{4}{7}$

Probability of getting a blue ball $P(B) = \frac{3}{7}$

Probability of getting red and blue ball

$P(A \cap B) = \frac{0}{7}$

Probability of getting either red or blue ⇒

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{4}{7} + \frac{3}{7} - \frac{0}{7} = 1$

Conditional Probability:-

Conditional Events:- If A, B are two events in a sample space then the event of happening B after of event A happening is called conditional Event.

It is denoted by $\frac{B}{A}$.

Conditional Probability:- If A, B are two events in a sample space S and $P(A) \neq 0$ then the probability of B after the event A has occured is called conditional probability of B given A.

→ It is denoted by $P(B/A)$

→ If A, B are 2 events in a sample space S such that $P(A) \neq 0$ then

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad P(A) > 0$$

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$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

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Random Variables:-

- If the numerical values assumed by a variable are the result of some chance factors, so that particular value can't be exactly predicted in advance then the variable is called a Random variable
- Random variables are denoted by X, Y, Z
- Random variables are of two types
  - → Continuous random variable
  - → Discrete random variable

Continuous random variable:-

A Continuous random variable is one which can be assumed any value within the interval i.e all values of continuous scale.

Ex:- The weight of group of individuals
The height of a group of individuals.

Discrete random variable:-

A discrete random variable is one which can assume only isolated value.

Ex:- The no. of heads in four tosses of a coin is a discrete random variable as it can't assume value other than 0, 1, 2, 3, 4

Discrete Probability distribution:-

Let a random variable 'X' assume values $x_1\ x_2\ x_3 \ldots x_n$ with probabilities $P_1\ P_2\ P_3 \ldots P_n$ where $P(X=x) = P_x \geq 0$ for each x_i and $P_1 + P_2 + P_3 + \cdots + P_n = \sum_{i=1}^{n} P_i = 1$ is called the discrete probability distribution for 'x'; a total probability ~~is~~ 1 is distributed over a several values of the random variables.

Mean and Variance of random variable:-

Let $X: x_1, x_2\ x_3\ x_4 \ldots x_n$ are random variables with probabilities $P_1\ P_2\ P_3 \ldots P_n$ be a discrete probability distribution, we denote the mean $\mu = \dfrac{\sum P_i x_i}{\sum P_i} = \sum P_i x_i \quad [\because \sum P_i = 1]$

Variance $\sigma^2 = \sum P_i (x_i - \mu)^2$

Standard deviation $= \sqrt{\text{Variance}}$

Note:-

→ $\int_a^b f(x)\,dx = 1$

→ Mean $\mu = \int_a^b x f(x)\,dx$

→ Median $= \int_a^M f(x)\,dx = \frac{1}{2}$

→ Mean $\mu = E(x) = \int x f(x)\,dx$

Variance $V(x) = E(x^2) - [E(x)]^2$

where $E(x^2) = \int x^2 f(x)\,dx$

→ Mean $= E(x) = \Sigma x\, p(x)$

Q) A random variable x has probabilitic density function, given by $f(x) = ax^2$, $2 \le x \le 3$. find a.

Sol:- Total probability = 1

$\Rightarrow \int_a^b f(x)\,dx = 1$

$\Rightarrow \int_2^3 ax^2\,dx = 1$

$\Rightarrow \left[a \frac{x^3}{3}\right]_2^3 = 1$

$\Rightarrow \frac{a}{3}[27 - 8] = 1$

$\Rightarrow \frac{a}{3}(19) = 1$

$19a = 3$

$a = \frac{3}{19}$ //

Q) A random variable x has the probability density function $f(x) = 6x(1-x)$, $0 \le x \le 1$. Find mean, median, mode and comment on their distribution.

Sol:- Mean $\mu = \int_a^b x f(x)\,dx$

$$\Rightarrow \int_0^1 x \cdot 6x(1-x)\,dx = \int_0^1 (6x^2 - 6x^3)\,dx$$

$$\Rightarrow \left[6\frac{x^3}{3} - 6\frac{x^4}{4}\right]_0^1 = \frac{6}{3} - \frac{6}{4} = \frac{24-18}{12} = \frac{1}{2}$$

$$\boxed{\text{Mean } \mu = \frac{1}{2}}$$

Median $M = \int_a^M f(x)\,dx = \frac{1}{2}$

$$\Rightarrow \int_0^M 6x(1-x)\,dx = \frac{1}{2}$$

$$\Rightarrow \int_0^M (6x - 6x^2)\,dx = \frac{1}{2}$$

$$\Rightarrow \left[\frac{6x^2}{2} - 6\frac{x^3}{3}\right]_0^M = \frac{1}{2}$$

$$\Rightarrow \frac{6M^2}{2} - \frac{6M^3}{3} - 0 = \frac{1}{2}$$

$$\Rightarrow \frac{18m^2 - 12m^3}{6} = \frac{1}{2}$$

$$\Rightarrow 6\left[\frac{3M^2 - 2m^3}{6}\right] = \frac{1}{2}$$

$$3M^2 - 2M^3 = \frac{1}{2}$$

$$4M^3 - 6M^2 + 1 = 0$$

$$M = \frac{1}{2}, \frac{1 \pm \sqrt{3}}{2}$$

$$\boxed{M = \frac{1}{2}, \quad 0 \le x \le 1}$$

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Mode is given $f'(x)=0$, $f''(x)<0$

$f(x) = 6x - 6x^2$

$f'(x) = 6 - 12x = 0$

$6 = 12x$

$x = \frac{1}{2}$

$f''(x) = -12 < 0$

$\boxed{\therefore \text{ Mode } = \frac{1}{2}}$

Therefore, Mean = Median = Mode = $\frac{1}{2}$

Hence it is a symmetric distribution.

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Q) A continuous random variable x has a probability density function $f(x) = kx^2$, $0 < x \leq 1$. Find the value of k, also obtain the mean of the variable x.

Sol:- Total probability = 1

$$\int_a^b f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 kx^2\,dx = 1$$

$$= \left[k\,\frac{x^3}{3}\right]_0^1 = 1$$

$$k \cdot \frac{1}{3} = 1 \Rightarrow \boxed{k = 3}$$

$$\text{Mean} = E(x) = \int_a^b x f(x)\, dx$$

$$\Rightarrow \int_0^1 x\, kx^2\, dx \Rightarrow \int_0^1 3x^3\, dx$$

$$\text{Mean} = E(x) = \left[\frac{3x^4}{4}\right]_0^1 = \frac{3}{4}$$

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8) A random variable has the following probability function values of

$X_{(x)}$	0	1	2	3	4	5	6	7
$P_{(x)}$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i) Find the value of k

ii) The value of $P(x<6)$

iii) The value of $P(x \geq 6)$

Sol:- we know that $\sum_{n=0}^{7} P(x) = 1$

$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$\Rightarrow 10k^2 + 9k - 1 = 0$$

$$k = \frac{1}{10}, -1$$

Therefore $k = \frac{1}{10}$ [$\because$ $P(x)$ can't be -ve]

ii) $P(x<6)$

$$\Rightarrow P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$$

$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2$$

$$\Rightarrow 8k + k^2 = \frac{8}{10} + \frac{1}{100} = \frac{81}{100}$$

iii) $P(x \geq 6)$

$\Rightarrow P(x=6) + P(x=7)$

$= 2k^2 + 7k^2 + k \Rightarrow 9k^2 + k$

$\Rightarrow 9\left(\frac{1}{10}\right)^2 + \frac{1}{10} \Rightarrow \frac{9}{100} + \frac{1}{10} = \frac{19}{100}$ //

Q) If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$, $P(A \cup B) = \frac{11}{12}$, then find $P(B/A)$

Sol:-

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A) + P(B) - P(A \cup B)}{P(A)}$$

$$\Rightarrow \frac{\frac{1}{3} + \frac{3}{4} - \frac{11}{12}}{\frac{1}{3}} \Rightarrow \frac{4+9-11}{\cancel{12}\,4} \times \cancel{3}$$

$$\Rightarrow \frac{2}{4} = \frac{1}{2}$$

Q) If $f(x) = \begin{cases} \frac{x+1}{2}, & \text{if } -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ where x has probability density function. Find the mean, variance and standard deviation of x.

Sol:- Mean $= E(x) = \int_a^b x\, f(x)\, dx = \int_{-1}^{1} \frac{x(x+1)}{2}\, dx$

$$\Rightarrow \frac{1}{2}\int_{-1}^{1}(x^2+x)\,dx \Rightarrow \frac{1}{2}\left[\frac{x^3}{3}+\frac{x^2}{2}\right]_{-1}^{1}$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{3}+\frac{1}{2}+\frac{1}{3}-\frac{1}{2}\right] = \frac{1}{2}\times\frac{2}{3} = \frac{1}{3}$$

Variance $V(x) = E(x^2) - [E(x)]^2$

$$E(x^2) = \int_{-1}^{1} x^2 f(x)\,dx = \int_{-1}^{1} x^2 \frac{(x+1)}{2}\,dx$$

$$\Rightarrow \frac{1}{2}\int_{-1}^{1}(x^3+x^2)\,dx = \frac{1}{2}\left[\frac{x^4}{4}+\frac{x^3}{3}\right]_{-1}^{1}$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{4}+\frac{1}{3}-\frac{1}{4}+\frac{1}{3}\right] \Rightarrow \frac{1}{2}\times\frac{2}{3} = \frac{1}{3}$$

Variance $V(x) = E(x^2) - [E(x)]^2 = \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$

standard deviation

$$\Rightarrow \sqrt{\text{variance}} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$$

Expectations

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1. $E(ax) = a\,E(x)$
2. $E(a) = a$
3. $V(ax) = a^2 V$
4. $V(x+b) = V(x)$
5. $V(ax+b) = a^2 V(x)$

Q) obtain the mean & variance of random variable Y given that $Y = 3X+4$ and X is a random variable with its mean is 6 & variance is 4.

Sol:- $Y = 3x+4$

$\boxed{\begin{array}{l} \text{Mean} - E(x) = 6 \\ \text{Variance} = V(x) = 4 \end{array}}$

$E(Y) = E(3x+4)$

$= E(3x) + E(4)$

$= 3\,E(x) + 4$

$= 3 \times 6 + 4 = 22$

$\boxed{E(Y) = 22}$

$V(Y) = V(3x+4)$ $\qquad V(ax+b) = a^2 V(x)$

$= 3^2\, V(x)$

$= 9 \times 4 = 36$

$\boxed{V(Y) = 36}$

Q) Prove that

$E(ax+b) = \cancel{ab(x)} + aE(x)+b$ is x is a random variable and a, b are constants.

$E(ax+b) = \sum_{i=1}^{n} (ax+b)\,p(x)$

$\Rightarrow \sum_{i=1}^{n} ax_i\,(px_i) + \sum_{i=1}^{n} b\,p(x_i)$

$\Rightarrow a \sum_{i=1}^{n} x_i\, p(x_i) + b \sum_{i=1}^{n} p(x_i)$ $\qquad \boxed{\because \sum_{i=1}^{n} P(x_i) = 1}$

$\Rightarrow a\,E(x) + b(1)$

$\Rightarrow a\,E(x) + b$

Q) Let X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax+3a, & 2 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

i) Find the value of a

ii) The value of $P(X \le 1.5)$

Sol :- Total probability $\int_a^b f(x)\,dx = 1$

$$\Rightarrow \int_0^3 f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx = 1$$

$$\Rightarrow \left[a\frac{x^2}{2}\right]_0^1 + \left[ax\right]_1^2 + \left[-a\frac{x^2}{2} + 3ax\right]_2^3 = 1$$

$$\Rightarrow a\left(\frac{1}{2}\right) + a(2-1) + \left[-\frac{9a}{2} + 9a + \frac{4a}{2} - 6a\right] = 1$$

$$\Rightarrow \frac{a}{2} + a - \frac{9a}{2} + 9a + \frac{4a}{2} - 6a = 1$$

$$\Rightarrow a + 2a - 9a + 18a + 4a - 12a = 2$$

$$\Rightarrow 4a = 2$$

$$\Rightarrow \boxed{a = \frac{1}{2}}$$

ii) $P(X \le 1.5) = \int_0^{1.5} f(x)\,dx$

$$\Rightarrow \int_0^1 f(x)\,dx + \int_1^{1.5} f(x)\,dx$$

$$\Rightarrow \int_0^1 ax\,dx + \int_1^{1.5} a\,dx$$

$$\Rightarrow \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{2}\left[x\right]_1^{1.5}$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}(1.5-1)$$

$$\Rightarrow \frac{1}{4} + \frac{1}{4} = \frac{1}{2} //$$

$$\boxed{P(X \le 1.5) = \frac{1}{2}}$$

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Conditional Probability:-

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Multiplication Theorem:-

$$P(A \cap B) = P(A) \cdot P(B/A) \quad ; \quad P(A) > 0$$

$$P(B \cap A) = P(B) \cdot P(A/B) \quad ; \quad P(B) > 0$$

Q] State and prove Bayes theorem.

statement:- If $E_1, E_2 ---- E_n$ are mutually exclusive and exhaustive events with $P(E_i) \neq 0$ $(i = 0, 1, 2, --- n)$ of the random experiment then for any arbitrary event A of the sample space S of the above experiment with $P(A) \geq 0$ we have

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}$$

Proof:- Let S be the sample space of the random experiment, the events $E_1, E_2 ---- E_n$ be exhaustive.

$$S = E_1 \cup E_2 \cup E_3 \cup ---- \cup E_n \quad [\because A \subseteq S]$$

$$A = A \cap S \Rightarrow A \cap [E_1 \cup E_2 \cup E_3 \cup ---- \cup E_n]$$

$$A = [(A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cup --- \cup (A \cap E_n)]$$

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3) + ----- + P(A \cap E_n)$$

By multiplication theorem.

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) P(A/E_3) + \cdots + P(E_n) P(A/E_n)$$

$$\therefore P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A/E_i) \quad -(1)$$

from Conditional probability

$$P(E_i/A) = \frac{P(A \cap E_i)}{P(A)} \quad -(2)$$

from multiplication Theorem

$$P(A \cap E_i) = P(E_i) \cdot P(A/E_i) \quad -(3)$$

Substitute (1) and (3) in (2)

$$P(E_i/A) = \frac{P(E_i) P(A/E_i)}{\sum_{i=1}^{n} P(E_i) P(A/E_i)}$$

Q) A bag X contains 2 white and 3 red balls. A bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bag and is found to be red. Find the probability that it was drawn from bag Y.

Sol:- Let E_1 is the ball drawn from bag X

Let E_2 is the ball drawn from bag Y

Let A be the red ball.

R.T.P:-

To find $P(E_2/A)$

By Bayes theorem

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

The 2 bags are equally likely to be selected

then $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{2}$

$P(A/E_1) = P(A$ red ball is drawn from bag $x) = \frac{3}{5}$

$P(A/E_2) = P(A$ red ball is drawn from bag $y) = \frac{5}{9}$

$$P(E_2/A) = \frac{\frac{1}{2}\cdot\frac{5}{9}}{\frac{1}{2}\cdot\frac{3}{5} + \frac{1}{2}\cdot\frac{5}{9}} = \frac{25}{52}$$

$$\boxed{P(E_2/A) = \frac{25}{52}}$$

Q) In a bolt factory machines A, B, C manufactures 25%, 35%, and 40% of the total of their out of 5%, 4%, 2% are defective bolts. A bolt is drawn at random from the product & found to be defective. what are the probabilities that it was manufactured by A, B, C,

Sol:- Let E_1, E_2 and E_3 denotes the events that the bolts are manufactured by machines A, B, C

$P(E_1) = 25\% = \frac{25}{100}$

$P(E_2) = \frac{35}{100}$, $P(E_3) = \frac{40}{100}$.

Let 'A' denotes the event that the bolt factory selects random bolt is defective.

$P(A/E_1) = \frac{5}{100}$, $P(A/E_2) = \frac{4}{100}$, $P(A/E_3) = \frac{2}{100}$.

By Bayes Theorem

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{\sum_{i=1}^{3} P(E_i)\cdot P(A/E_i)}$$

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$\Rightarrow \frac{\frac{25}{100}\cdot\frac{5}{100}}{\frac{25}{100}\cdot\frac{5}{100} + \frac{35}{100}\cdot\frac{4}{100} + \frac{40}{100}\cdot\frac{2}{100}} = \frac{25}{69} =$$

$P(E_2/A) = \frac{28}{69} = 0.405$

$P(E_3/A) = \frac{16}{69} = 0.231$

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Q) The chances of X, Y, Z becoming manager of a certain company are 4:2:3. The probability that a bonous scheme is introduced if X, Y, Z becomes manager are 0.3, 0.5 and 0.8 if the bonus scheme has been introduced, what is the probability that X, Y, Z are appointed as managers.

Sol:- $X:Y:Z = 4:2:3$

Total ratio $= 4+2+3 = 9$

$P(E_1) = \frac{4}{9}$, $P(E_2) = \frac{2}{9}$, $P(E_3) = \frac{3}{9}$

$P(A/E_1) = 0.3 = \frac{3}{10}$, $P(A/E_2) = 0.5 = \frac{5}{10}$, $P(A/E_3) = 0.8 = \frac{8}{10}$

The probability that X is appointed as manager is

By Bayes Theorem:

$$P(E_1/A) = \frac{P(E_i)\cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\cdot P(A/E_i)}$$

$$\Rightarrow \frac{\frac{4}{9}\times\frac{3}{10}}{\frac{4}{9}\times\frac{3}{10}+\frac{2}{9}\times\frac{5}{10}+\frac{3}{9}\times\frac{8}{10}} = \frac{6}{23}$$

The probability that Y is appointed as manager is

$$P(E_2/A) = \frac{\frac{2}{9}\times\frac{5}{10}}{\frac{4}{9}\times\frac{3}{10}+\frac{2}{9}\times\frac{5}{10}+\frac{3}{9}\times\frac{8}{10}} = \frac{5}{23}$$

The probability that Z is appointed as manager is

$$P(E_3/A) = \frac{\frac{3}{9}\times\frac{8}{10}}{\frac{4}{9}\times\frac{3}{10}+\frac{2}{9}\times\frac{5}{10}+\frac{3}{9}\times\frac{8}{10}} = \frac{12}{23}$$

Q) A random variable X has the following probability function

$X=x$:	0	1	2
$P(X=x)$:	K	2K	3K

Find i) $P(x<2)$ ii) $P(x \geq 2)$ iii) $P(0<x<2)$

Sol:-

$X=x$:	0	1	2
$P(X=x)$:	K	2k	3K

$\sum_{x=0}^{2} P(x)=1$ [$\because$ Total probability $=1$]

$K+2K+3K=1 \Rightarrow 6K=1 \Rightarrow \boxed{K=\frac{1}{6}}$

i) $P(x<2)$: $P(X=0)+P(X=1)$

$\Rightarrow K+2K \Rightarrow 3K \Rightarrow 3\times\frac{1}{6}=\frac{1}{2}$

ii) $P(x\geq 2)$: $P(X=2)$

$\Rightarrow 3K \Rightarrow 3\times\frac{1}{6}=\frac{1}{32}$

iii) $P(0<X<2)$:- $P(X=1)$

$\Rightarrow 2K=2\times\frac{1}{6}=\frac{1}{3}$

Q) If x is a random variable with the following distribution

x :	0	1	2	3	4	5	6
$P(x)$:	0.1	0.15	0.2	0.3	0.1	0.05	0.1

Find $E(x)$, $E(x-2)$, $E(3x+1)$

Sol :-

$E(x) = \Sigma x\, p(x)$

$\Rightarrow 0\times 0.1 + 1\times 0.15 + 2\times 0.2 + 3\times 0.3 + 4\times 0.1 + 5\times 0.05 + 6\times 0.01$

$\boxed{E(x) \Rightarrow 2.7}$

$E(x-2) = E(x) - E(2)$

$\Rightarrow 2.7 - 2 = 0.7$

$\boxed{E(x-2) = 0.7}$

$E(3x+1) = 3E(x) + E(1)$

$\Rightarrow 3\times 2.7 + 1 = 9.1$

$\boxed{E(3x+1) = 9.1}$

Q) If X is a random variable in the following distribution

x :	1	3	4	5
$P(x)$:	0.4	0.1	0.2	0.3

Find the mean, variance & standard deviation of x.

Sol :-

Mean :- $E(x) = \sum_{i=1}^{5} x f(x)$

$\Rightarrow 1\times 0.4 + 3\times 0.1 + 4\times 0.2 + 5\times 0.3$

$\Rightarrow 3$

$E(x^2) = \sum x^2 f(x)$

$\Rightarrow 1^2 \times 0.4 + 3^2 \times 0.1 + 4^2 \times 0.2 + 5^2 \times 0.3$

$E(x^2) \Rightarrow 12$

Variance $V(x) = E(x^2) - [E(x)]^2$

$\Rightarrow 12 - 3^2 \Rightarrow 12 - 9 = 3$

standard deviation

$\Rightarrow \sqrt{\text{variance}} = \sqrt{3}$.

Q) If X is a random variable in the following distribution

x:	0	1	2	3	4	5	6
$P(x)$:	0.15	0.1	0.05	0.3	0.2	0.1	0.1

find $E(X)$, $E(X-4)$, $E(2X+1)$, $V(X)$, $V(2X+3)$

Sol :- $E(X) = \sum_{i=0}^{6} x P(x)$

$\Rightarrow 0 \times 0.15 + 1 \times 0.1 + 2 \times 0.05 + 3 \times 0.3 + 4 \times 0.2 + 5 \times 0.1 + 6 \times 0.1$

$\boxed{E(X) = 3}$

$E(x^2) = \sum x^2 f(x)$

$\Rightarrow 1^2 \times 0.1 + 2^2 \times 0.05 + 3^2 \times 0.3 + 4^2 \times 0.2 + 5^2 \times 0.1 + 6^2 \times 0.1$

$\boxed{E(x^2) = 12.3}$

$E(X-4) = 3 - E(4)$

$\Rightarrow 3 - 4 = -1$

$E(2X+1) = 2E(X) + E(1)$

$\Rightarrow 2 \times 3 + 1 = 7$

$V(X) = E(x^2) - [E(x)]^2$

$= 12.3 - 9 = 3.3$

$V(2X+3) =$

$2^2(3.3) = 4(3.3)$

$= 13.2$

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Moments:-

Note:- Moments about the mean or central moments ($\mu_1, \mu_2, \mu_3, \mu_4$)

→ Moments about the origin or raw moments ($\mu_1', \mu_2', \mu_3', \mu_4'$)

(or)

Moments about any point $x = A$ (raw moments) ($\mu_1', \mu_2', \mu_3', \mu_4'$)

→

i) $\mu_1 = 0$ (Always)

ii) $\mu_2 = \mu_2' - \mu_1'^2$

iii) $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

iv) $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

Q) The first four moments of a random variable about origin are 1, 10, 20, 35. Find the Central moments.

Sol:- Moments about origin are

$\mu_1' = 1, \quad \mu_2' = 10, \quad \mu_3' = 20, \quad \mu_4' = 35$

Central moments

$\mu_1 = 0$ (Always)

$\mu_2 = \mu_2' - \mu_1'^2 \Rightarrow 10 - 1 = 9$

$\boxed{\mu_2 = 9}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$\Rightarrow 20 - 3(10)(1) + 2(1)^3 = -8$

$\boxed{\mu_3 = -8}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$\Rightarrow 35 - 4(20)(1) + 6(10)(1)^2 - 3(1)^4$

$\boxed{\mu_4 = 12}$

Therefore, Central moments

$\mu_1 = 0, \ \mu_2 = 9, \ \mu_3 = -8, \ \mu_4 = 12.$

Q) If the first 4 moments about $x = 4$ are $1, -4, 12, 24$. find the corresponding about mean.

Sol:- $\mu_1' = 1, \ \mu_2' = -4, \ \mu_3' = 12, \ \mu_4' = 24.$

$\mu_1 = 0$ (Always)

$\mu_2 = \mu_2' - \mu_1'^2 = -4 - (1)^2 = -5$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = 12 - 3(-4)(1) + 2(1)^3 = 26$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$\Rightarrow 24 - 4(12)(1) + 6(-4)(1)^2 - 3(1)^4 = -51$

Therefore, moments about mean are

$\mu_1 = 0, \ \mu_2 = -5, \ \mu_3 = 26, \ \mu_4 = -51.$

Q] The first 4 moments of a random variable about $x=1$ are 5, 20, 40, 120. Find the first 4 central moments & also the moment at $x=4$.

Sol:- $\mu_1'=5,\ \mu_2'=20,\ \mu_3'=40,\ \mu_4'=120.$

$\mu_1=0$

$\mu_2=\mu_2'-\mu_1'^2 \Rightarrow 20-25=-5$

$\mu_3=\mu_3'-3\mu_2'\mu_1'+2\mu_1'^3$

$\Rightarrow 40-3(20)(5)+2(125)$

$\Rightarrow 40-300+250=-10$

$\mu_4=\mu_4'-4\mu_3'\mu_1'+6\mu_2'\mu_1'^2-3\mu_1'^4$

$=120-4(40)(5)+6(20)(25)-3(625)$

$\Rightarrow 445$

$\boxed{\mu_1=0}$ $\boxed{\mu_2=-5}$ $\boxed{\mu_3=-10}$ $\boxed{\mu_4=445}$

Since $\bar{x}=\mu_1'+A \Rightarrow 5+1=6$

The moment at $x=4 \Rightarrow \boxed{A=4}$

$\bar{x}=\mu_1'+A$

$\mu_1'=\bar{x}-A \Rightarrow 6-4=2$

$\mu_1'=2$

$\mu_2'=\mu_2+\mu_1'^2 \Rightarrow -5+2^2 \Rightarrow \boxed{\mu_2'=-1}$

$\mu_3' = \mu_3 + 3\mu_2'\mu_1' - 2\mu_1'^3$

$\Rightarrow -10 + 3(-1)(2) - 2(2)^3$

$\boxed{\mu_3' \Rightarrow -32}$

$\mu_4' = \mu_4 + 4\mu_3'\mu_1' - 6\mu_2'\mu_1'^2 + 3\mu_1'^4$

$\mu_4' \Rightarrow 445 + 4(-32)(2) - 6(-1)(2)^2 + 3(2)^4$

$\boxed{\mu_4' = 261}$

Q) Calculate the first 4 moments of the following distribution about the mean (central moments). Hence find β_1, β_2

x :	0	1	2	3	4	5	6	7	8
f :	1	8	28	56	70	56	28	8	1

Sol :-

x	f	$d = x-4$	fd	fd^2	fd^3	fd^4
0	1	-4	-4	16	-64	256
1	8	-3	-24	72	-216	648
2	28	-2	-56	112	-224	448
3	56	-1	-56	56	-56	56
4	70	0	0	0	0	0
5	56	1	56	56	56	56
6	28	2	56	112	224	448
7	8	3	24	72	216	648
8	1	4	4	16	64	256
$\Sigma x = 36$	$\Sigma f = N = 256$		$\Sigma fd = 0$	$\Sigma fd^2 = 512$	$\Sigma fd^3 = 0$	$\Sigma fd^4 = 2816$

$\bar{x} = \frac{\Sigma x}{n} = \frac{36}{9} = 4$

$\mu_1' = \frac{\Sigma fd}{N} = \frac{0}{256} = 0$

$\mu_2' = \frac{\Sigma fd^2}{N} = \frac{512}{256} = 2$

$\mu_3' = \frac{\Sigma fd^3}{N} = \frac{0}{256} = 0$

$\mu_4' = \frac{\Sigma fd^4}{N} = \frac{2816}{256} = 11$

Moments about the mean

$\mu_1 = 0$ (Always)

$\mu_2 = \mu_2' - \mu_1'^2 = 2 - 0^2 = 2$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_3 \Rightarrow 0 - 3 \times 2 \times 0 + 2 \times 0 = 0$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$\mu_4 = 11$.

skewness And kurtosis

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{0}{2^3} = 0$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{11}{4}$

If $\beta_1 = 0$, the symmetric distribution.